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that if all seven escribed spheres are present the configuration can always be placed in one of these two types.

If one, two, or three of the outer spheres are lacking it will be found that we can place the configuration in *either* of the two types named above according as we consider the centers of the spheres which are at infinity to be *above* or *below* the vertex in question. These special cases occupy a limiting position between the two main types.

It may be pointed out that each vertex of the tetrahedron is collinear with four pairs of the eight points I, E_1, E_2, E_3, E_7, E_4, E_5, E_6. Fig. 1 shows the second type of tetrahedron; the figure that accompanies Geom. 417 shows the first type.

Editorial Note.—The discussion, scarcely intuitional, is printed for the division of the general case (7 finite ex-centers) into two sub-types. The intuitional view-point for the division into types with 4, 5, 6, 7 escribed spheres desired by the proposer is afforded by considering the angles at opposite pairs of edges. If equal the corresponding ex-center is at infinity, if unequal the ex-center is in the compartment of greater angle.

436. Proposed by A. J. KEMPNER, University of Illinois.

Given in a plane two similar curves arbitrarily situated, except that they shall possess the same sense of direction (which, of course, does not mean that they shall be similarly located). Let corresponding points on both curves be joined by straight lines, and let all of these straight lines be divided in the same ratio $\lambda : 1$, λ being any real number. Prove that the points of division all lie on a curve similar to the two given curves except when they all happen to fall together.

SOLUTION BY H. T. BIGELOW, Lafayette, Indiana.

Let the parametric equations of one of the curves be

$$x_1 = f(t), \qquad y_1 = \varphi(t). \tag{1}$$

The second curve, by reason of the similarity, is derivable from the first by an expansion from the origin, a rotation about the origin, and a translation. Its equation is, therefore,

$$\begin{aligned} x_2 &= a + k \cos \vartheta \cdot f(t) - k \sin \vartheta \cdot \varphi(t), \\ y_2 &= b + k \sin \vartheta \cdot f(t) + k \cos \vartheta \cdot \varphi(t), \end{aligned} \tag{2}$$

and corresponding points on the two curves (1) and (2) are given by equal values of t.

The corresponding point on the third curve is given by the equations

$$x_3 = \frac{x_1 + \lambda x_2}{1 + \lambda}, \qquad y_3 = \frac{y_1 + \lambda y_2}{1 + \lambda};$$

and

$$x_3 = \frac{\lambda a}{1 + \lambda} + \frac{(1 + \lambda k \cos \vartheta)}{1 + \lambda} f(t) - \frac{\lambda k \sin \vartheta}{1 + \lambda} \varphi(t),$$

$$y_3 = \frac{\lambda b}{1 + \lambda} + \frac{\lambda k \sin \vartheta}{1 + \lambda} f(t) + \frac{1 + \lambda k \cos \lambda}{1 + \lambda} \varphi(t).$$

If now we set $a' = \lambda a/(1 + \lambda)$ and $b' = \lambda b/(1 + \lambda)$ and choose k' and ϑ' so as to make $k' \cos \vartheta' = (1 + \lambda k \cos \vartheta)/(1 + \lambda)$, $k' \sin \vartheta' = \lambda k \sin \vartheta/(1 + \lambda)$, the equations of the third curve become

$$\begin{aligned} x_3 &= a' + k' \cos \vartheta' \cdot f(t) - k' \sin \vartheta' \cdot \varphi(t), \\ y_3 &= b' + k' \sin \vartheta' \cdot f(t) + k' \cos \vartheta' \cdot \varphi(t). \end{aligned} \tag{3}$$

These equations are of the form (2) and consequently represent a curve similar to (1) and (2).

437. Proposed by J. BROOKS SMITH, Hampden Sidney, Va.

Let D, E, F be three arbitrary points taken on the sides of a triangle ABC. If Δ and Δ' be the areas of the triangles ABC and DEF, show that

$$\frac{\Delta'}{\Delta} = \frac{AF \cdot BD \cdot CE + AE \cdot CD \cdot BF}{abc},$$

the sign of each factor being determined as follows: Each segment adjacent to one of the vertices of the triangle ABC is to be regarded as positive or negative according as it is drawn towards or from the other vertex of the side containing the segment.

Solution by Paul Capron, Annapolis, Md.

Let $AF = m$, $AE = q$, $BF = r$, $BD = k$, $CD = p$, $CE = l$; and let the areas of the triangles be $AFE = \Delta_1$, $BDF = \Delta_2$, $CED = \Delta_3$. Then, a, b, c being positive we have, in accordance with the given convention of signs, and the further convention that the area of a Δ shall be positive if the cyclic arrangement of its vertices is contra-clockwise, negative otherwise:

$$\frac{\Delta_1}{\Delta} = \frac{mq}{bc}, \qquad \frac{\Delta_2}{\Delta} = \frac{kr}{ca}, \qquad \frac{\Delta_3}{\Delta} = \frac{lp}{ab};$$

$$\frac{\Delta - \Delta_1 - \Delta_2 - \Delta_3}{\Delta} = \frac{\Delta'}{\Delta} = \frac{abc - amq - bkr - clp}{abc}.$$

Substituting $a = k + p$, $b = l + q$, $c = m + r$, we have

$$\frac{\Delta'}{\Delta} = \frac{klm + pqr}{abc} = \frac{AF \cdot BD \cdot CE + AE \cdot CD \cdot BF}{abc}.$$

Also solved by S. W. Reaves, H. C. Feemster, C. N. Schmall, Horace Olson, and the Proposer.

438. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

By means of the theorem that the product of the diagonals of a quadrilateral inscribed in a circle is equal to the sum of the two products of pairs of opposite sides, obtain the usual formulas for $\sin(\alpha \pm \beta)$ and $\cos(\alpha \pm \beta)$ in terms of $\sin \alpha$, $\sin \beta$, $\cos \alpha$, $\cos \beta$.

Solution by A. M. Harding, University of Arkansas.

Suppose PR is a diameter and $\angle SPR = \alpha$ and $\angle RPQ = \beta$. (Fig. 1.) Then

$$PR \cdot SQ = PS \cdot RQ + PQ \cdot SR$$

or

$$2r \cdot 2r \sin(\alpha + \beta) = 2r \cos \alpha \cdot 2r \sin \beta + 2r \cos \beta \cdot 2r \sin \alpha.$$